Hiroshi Ono's 17 stage order 10 Runge-Kutta scheme

See: A Runge-Kutta method of order 10 which minimizes truncation error, by Hiroshi Ono, Ibaraki University, The Japan Society for Industrial and Applied Mathematics, Vol. 13, No. 1, 2003, pp 35 - 44.

The following rational approximations for the nodes suggested by Hiroshi Ono are used in the scheme considered here.

$$c_2 = \frac{13705}{40819},\ c_3 = \frac{15707}{29841},\ c_{15} = \frac{15707}{29841},\ c_{16} = \frac{13705}{40819},\ c_{17} = 1.$$

The value for the weight b_6 (and therefore also b_{14}) is also a rational approximation for the value suggested by Hiroshi Ono.
The values for the weights b_2, b_3 and b_7 (and therefore also b_{16}, b_{15} and b_{13}) are not those suggested by Hiroshi Ono but are chosen to minimize the 2-norm of the linking coefficients. This does not affect the other characteristics of the scheme.

$$b_2 = -\frac{13}{593},\ b_3 = -\frac{30}{529},\ b_4 = 0,\ b_5 = 0,\ b_6 = -\frac{19}{339},\ b_7 = \frac{107}{598},\ b_8 = 0,\ b_{13} = -\frac{107}{598},\ b_{14} = \frac{19}{339},\ b_{15} = \frac{30}{529},\ b_{16} = \frac{13}{593}.$$

The nodes:

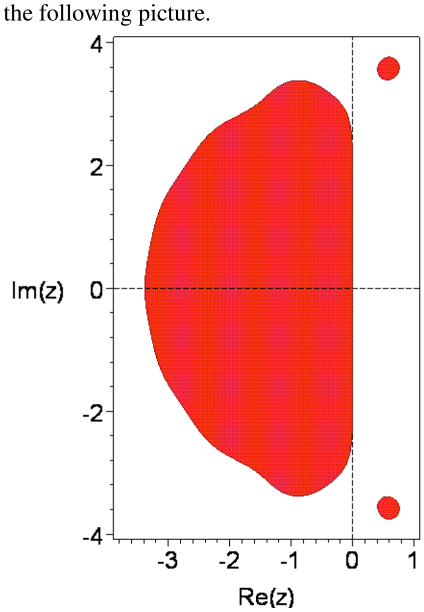
$$c_9 = \frac{1}{2} - \frac{\sqrt{147 - 42\sqrt{7}}}{42},\ c_{10} = \frac{1}{2} + \frac{\sqrt{147 + 42\sqrt{7}}}{42},\ c_{11} = \frac{1}{2} + \frac{\sqrt{147 - 42\sqrt{7}}}{42},\ c_{12} = \frac{1}{2} - \frac{\sqrt{147 + 42\sqrt{7}}}{42}$$

are the zeros of the derivative $P'_5(x) = \frac{d}{dx} P_5(x)$ of the **Legendre polynomial** $P_5(x)$ mapped linearly from the interval $[-1, 1]$ to the interval $[0, 1]$.
They provide nodes for **Gauss-Lobatto integration** on the interval $[0, 1]$.
These values give rise to the following weights:

$$b_1 = \frac{1}{30},\ b_9 = \frac{7}{30} + \frac{\sqrt{7}}{60},\ b_{10} = \frac{7}{30} - \frac{\sqrt{7}}{60},\ b_{11} = \frac{7}{30} + \frac{\sqrt{7}}{60},\ b_{12} = \frac{7}{30} - \frac{\sqrt{7}}{60},\ b_{17} = \frac{1}{30}.$$

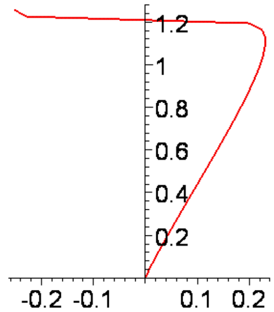
The principal error norm, that is, the 2-norm of the principal error terms is: $0.1252657404 \times 10^{(-5)}$.
The maximum magnitude of the linking coefficients is: 1.376373544.

The stability region for the scheme is shown in the following picture.



The real stability interval of the scheme is $[-3.3816, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval [0, 1.2017].

The coefficients correct to 85 digits are as follows.
c[2]=.3357505083417036184129939488963472892525539577157696170900805997207182929518116563365,
c[3]=.5263563553500217821118595221339767434067222948292617539626688113669112965383197614021,
c[4]=.7895345330250326731677892832009651151100834422438926309440032170503669448074796421031,
c[5]=.1852155685265047076970656199437875208987680209449849941608024849050604421517806064228,
c[6]=.2895345330250326731677892832009651151100834422438926309440032170503669448074796421031,
c[7]=.7659879027055932401898717206953675811356898851157703878511505293452324618973555436361,
c[8]=.1080739009578824490100240661758266719014599665988034811870817465871751958605805741080,
c[9]=.3573842417596774518429245029795604640404982636367873040901247917361510345429002009092,
c[10]=.8825276619647323464255014869796690751828678442680521196637911779185276585194132570617,
c[11]=.6426157582403225481570754970204395359595017363632126959098752082638489654570997990908,
c[12]=.1174723380352676535744985130203309248171321557319478803362088220814723414805867429383,
c[13]=.7659879027055932401898717206953675811356898851157703878511505293452324618973555436361,
c[14]=.2895345330250326731677892832009651151100834422438926309440032170503669448074796421031,
c[15]=.5263563553500217821118595221339767434067222948292617539626688113669112965383197614021,
c[16]=.3357505083417036184129939488963472892525539577157696170900805997207182929518116563365,
c[17]=1.,

a[2,1]=.3357505083417036184129939488963472892525539577157696170900805997207182929518116563365,
a[3,1]=.1137717040478783056449396184425018992536986324028992632761012508174661576282091865815,
a[3,2]=.4125846513021434764669199036914748441530236624263624906865675605494451389101105748206,
a[4,1]=.1973836332562581682919473208002412787775208605609731577360008042625917362018699105258,
a[4,2]=0.,
a[4,3]=.5921508997687745048758419624007238363325625816829194732080024127877752086056097315774,
a[5,1]=.1360001717992528326665261557210531883399922632350163155393510462488093550801937826726,
a[5,2]=0.,
a[5,3]=.8247208052028112223790799775745048180379960003630910417779053602715946236310693542837e-1,
a[5,4]=-.3325668379302924720736853353471614924502384232634042555633909737090837529152011167815e-1,
a[6,1]=.6546785199481110579756630335868870724165088583024509548938434492848601278421108871818e-1,
a[6,2]=0.,
a[6,3]=0.,
a[6,4]=.6858715620423033993966826640550281985093888681236135859756994188871385539662876478859e-3,
a[6,5]=.2233808094681792639708262971782213796699231675455239218686431727029937934693022657371,
a[7,1]=.2379439999135994223360182301765245366877534054337310606780367192080205084533052094979,
a[7,2]=0.,
a[7,3]=0.,
a[7,4]=.1285793626493546102687479560301629068933431333159680850577212924587597635005521433930,
a[7,5]=-.7303763776380165799305971651206183012312272113455920027358381669467410091843571268521,
a[7,6]=1.129840917780655787515702699609298438785820557711663244851230684625193199127855317597,
a[8,1]=.6062780896441720418288517632894493561013839292138719356621888281245740094338544165775e-1,
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a[8,3]=0.,
a[8,4]=0.,
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a[9,1]=.31044158654387218890840975388199959303584798683522008935916449211248286670056865369 20e-1,
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a[9,3]=0.,
a[9,4]=0.,
a[9,5]=0.,
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a[9,7]=.11176385413840843029896873382715056426616639184170370710417528015539697159707221556 09e-3,
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a[10,4]=0.,
a[10,5]=0.,
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a[10,8]=.25594267771708049411675905016341942252036200202345618631543262619891413892681247876 52,
a[10,9]=.71891127903498454375625048072704048066702616375794750446311235832232491249639377901 42,
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a[14,3]=0.,

a[14,4]=.68587156204230339939668266405502819850938886812361358597569941888713855396628764788859e-3,
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a[14,6]=-.354315266959561471543807477004472824842463468241712091672601415035879706466084241921 8,
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a[15,9]=0.,
a[15,10]=0.,
a[15,11]=0.,
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a[16,1]=.33575050834170361841299394889634728925255395771576961709008059972071829295181165633 65,
a[16,2]=0.,
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a[16,4]=0.,
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a[17,1]=.35284009145390657576953742500205269837920420947787066335740485751385983954453754599 64e-1,
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a[17,3]=-.51852530257519117008153543706583682146534040209234614132049270079074073447677587105 02,
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a[17,9]=-.20281215249997183418132083040210459236750611504086744891103388147985187342781698651 66,
a[17,10]=.40187344425260456741541924679952524080368182470696313611978997960788971506804660962 92,
a[17,11]=.69828086812384875164815945422548055525105417942719976326409213073560934000751205054 80,
a[17,12]=.27459533401274604136301429200265465257883379132782521576853981228193489572907063958 25,
a[17,13]=-.80710511588132885319511867792781040567320844078044038570021311684929854199099051009 04,
a[17,14]=.29864698833018538074805317741552351355992067693287699141734378044342982408535147787 70,
a[17,15]=.51852530257519117008153543706583682146534040209234614132049270079074073447677587105 02,
a[17,16]=.43685885623395546175199386160273955681535237182268102754615272531015154426440717374 16,

b[1]=.3333333333333333333333333333333333333333333333333333333333333333333e-1,
b[2]=-.21922428330522765598650927487352445193929173693086003372681281618887015177065767284 99e-1,

b[3]=-.5671077504725897920604914933837429111531190926275992438563327032136105860113421550095e-1,
b[4]=0.,
b[5]=0.,
b[6]=-.5604719764011799410029498525073746312684365781710914454277286135693215339233038348083e-1,
b[7]=.1789297658862876254180602006688963210702341137123745819397993311036789297658862876254,
b[8]=0.,
b[9]=.2774291885177431765083602625606543404285043197180408363394722409866844803871713937960,
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b[11]=.2774291885177431765083602625606543404285043197180408363394722409866844803871713937960,
b[12]=.1892374781489234901583064041060123262381623469486258303271944256799821862794952728707,
b[13]=-.1789297658862876254180602006688963210702341137123745819397993311036789297658862876254,
b[14]=.5604719764011799410029498525073746312684365781710914454277286135693215339233038348083e-1,
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b[16]=.2192242833052276559865092748735244519392917369308600337268128161888701517706576728499e-1,
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